ΑΣΚΗΣΕΙΣ (Αντίστροφες Τριγωνομετρικές)

1) Δίνεται $f(x)=\text{Arc}\eta\mu(|x|-3)$, Να βρείτε το πεδίο ορισμού της f

ΛΥΣΗ

Πρέπει, $\Delta(f)=\{x\in\mathbb{R}: -1\leq |x|-3\leq 1 \Rightarrow 2\leq |x|\leq 4\}=$

$=\{x\in\mathbb{R}: x\geq 2 \text{ ή } x\leq -2 \ \underline{\text{και}}\ x\geq -4 \text{ και } x\leq 4\}=$

$=[-4,-2]\cup[2,4]$.

2) Δίνεται $f(x)=\log\left(\text{Arc}\eta\mu\left(\frac{x+2}{5-x}\right)\right)$, Ποιο το πεδίο ορισμού της f;

ΛΥΣΗ

Πρέπει, $\Delta(f)=\left\{x\in\mathbb{R}: \text{Arc}\eta\mu\left(\frac{x+2}{5-x}\right)>0 \ \underline{\text{και}}\ \left|\frac{x+2}{5-x}\right|\leq 1\right\}=$

$=\left\{x\in\mathbb{R}: \frac{x+2}{5-x}>0 \ \underline{\text{και}}\ -1\leq\frac{x+2}{5-x}\leq 1\right\}=$

$=\left\{x\in\mathbb{R}: (x+2)(5-x)>0 \ \underline{\text{και}}\ \frac{x+2}{5-x}\geq -1 \text{ και } \frac{x+2}{5-x}\leq 1\right\}$

• $(x+2)(5-x)>0$ $\quad\underline{\text{και}}\quad$ • $\frac{x+2}{5-x}\geq -1$, $\forall x\neq 5 \Rightarrow \frac{x+2}{5-x}+1\geq 0\Rightarrow$

	-2		5	
$-$	$\circ$	$+$	$\circ$	$-$

$\Rightarrow \frac{\cancel{x}+2+5-\cancel{x}}{5-x}\geq 0 \Rightarrow \frac{7}{5-x}\geq 0\Rightarrow$

$\Rightarrow 7(5-x)\geq 0$, $\forall x\neq 5$

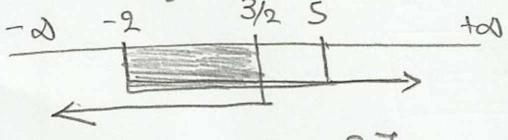
	5	
$+$	$\circ$	$-$

και • $\frac{x+2}{5-x}\leq 1 \Rightarrow \frac{x+2}{5-x}-1\leq 0\Rightarrow \frac{x+2-5+x}{5-x}\leq 0\Rightarrow$

$\Rightarrow \frac{2x-3}{5-x}\leq 0 \Rightarrow (2x-3)(5-x)\leq 0$

	$\frac{3}{2}$		5	
$-$	$\circ$	$+$	$\circ$	$-$

$\Delta(f)=\left\{x\in\mathbb{R}: x\in(-2,5)\ \underline{\text{και}}\ x\in(-\infty,5) \text{ και } x\in\left(-\infty,\frac{3}{2}\right]\cup(5,+\infty)\right\}$



Άρα $\Delta(f)=\left(-2,\frac{3}{2}\right]$

3) ΝΔΟ $2\cdot Arc\,\varepsilon\varphi \dfrac{1-\sqrt{1-x^2}}{x} = Arc\cdot\eta\mu\, x$, $0<|x|<1$

ΛΥΣΗ

Θέτουμε $Arc\,\varepsilon\varphi \dfrac{1-\sqrt{1-x^2}}{x} = \alpha \Rightarrow \boxed{\varepsilon\varphi\,\alpha = \dfrac{1-\sqrt{1-x^2}}{x}}$

Θέτουμε $Arc\,\eta\mu\, x = \beta \Rightarrow \boxed{\eta\mu\,\beta = x}$

ΘΔΟ $2\alpha = \beta \Rightarrow \eta\mu(2\alpha) = 2\cdot\sigma\upsilon\nu\alpha\,\eta\mu\alpha = 2\cdot\sigma\upsilon\nu^2\alpha\,\dfrac{\eta\mu\alpha}{\sigma\upsilon\nu\alpha} =$

$= 2\,\varepsilon\varphi\alpha\cdot\sigma\upsilon\nu^2\alpha = 2\varepsilon\varphi\alpha\,\dfrac{1}{1+\varepsilon\varphi^2\alpha} =$

$= 2\,\dfrac{1-\sqrt{1-x^2}}{x}\cdot\dfrac{1}{1+\left(\dfrac{1-\sqrt{1-x^2}}{x}\right)^2} =$

$= \dfrac{2-2\sqrt{1-x^2}}{x\left(1+\left(\dfrac{1-\sqrt{1-x^2}}{x}\right)^2\right)} = \ldots = x = \eta\mu\,\beta.$